Multi-boson correlations using wave-packets II.

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Abstract. We investigate the analytically solvable pion-laser model, and its generalization to arbitrary multiplicity distributions. Although this kind of extension of the model is possible, the pion laser model in its original form is unique: it is the only model in its class that posesses an analytic solution.

This work is dedicated to the memory of the late J. Zimányi and corresponds to a follow-up work on one of his favourite topics: the analytic solution of the pion laser model.

1 Introduction

Correlation between different identical particles from a thermalized source is due to the bosonic or fermionic nature of them. The effect was discovered by R. Hanbury Brown and Twiss in 1954, they pointed out that the observed correlation of photons carry information on the angular diameter of the emitting distant star [1]. In experimental high-energy physics, the phenomenon is known as GGLP effect [2], one commonly uses the term HBT-effect for the fact, that boson correlations appear and the shape of the correlation function is related to the geometry of the source. Since its discovery the HBT effect was proven to be an important tool in high energy physics for investigating the space-time extent of the reaction process.

In heavy ion physics, experiments usually measure charged pion correlations. If one neglects the final state interaction (FSI) and multi-particle correlation effects, the description of the correlation is simple: the correlation function is essentially the Fourier transform of the emission source distribution, referred to as source function. But in realistic situations neither FSI nor multi-particle correlations are negligible. Two-particle FSI can be taken into account either by generalized Coulomb corrections or by using the so-called imaging method [3]. Multi-particle correlation means that the final state outgoing wave-function needs to be symmetrized in all variables. This obviously requires $n!$ operation, where n is the number of particles. If the phase-space density of produced pions increases, then the effects arising from multi-particle symmetrization become significant, so at first sight this is a non-polynomially (NP) hard problem. Note that the computing time of NP hard problems of the order "n" increases faster than any power of n. Hence these problems for large n are essentially unsolvable numerically.

But in fact, there are such models in which one can overcome this difficulty and calculate the correlation functions using fully symmetrized wave-functions. The first success in this direction was achieved by S. Pratt [6], who showed that in a special case of source functions and factorization, the NP-hard problem reduces to a set of recurrence relations, called ring algebra, detailed first in ref. [7]. The next step were refs. [4,5], where the model was generalized to wave-packet states instead of plane-wave final states, and the recurrence relations were solved. The

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model had only one flaw: the multiplicity distribution of it is very special, and in some sense, unphysical. For example, coherent behaviour corresponds to Poisson multiplicity distribution in case of optical the optical lasers, while fully chaotic, rare gas limit corresponds to the Poisson multiplicity distribution in case of the pion laser model. In the present work we investigated if the original model of refs. [4,5,6,7] can be modified in such a way, that the Poisson multiplicity distribution will characterize the pion condensate, and not the chaotic state.

In sections 2, 3, 4 and 5 we recapitulate and slightly re-formulate earlier results of refs. [4,5,6,7] to prepare their generalization, utilizing the idea of Poisson transformation. Section 6 describes the outcome of this investigation and Section 7 summarizes the results.

2 Definition of the model

The investigated pion-laser model describes a multi-particle system containing arbitrary number of bosonic wave-packet states. The notations ξ, χ and σ stand for the center in coordinate space, the center in momentum space and the width in momentum space of a given wave packet, respectively, and $\alpha := (\xi, \chi, \sigma)$ is an abbreviated notation for these parmeters (for example, integrating over α means integration over all variables). The quantities ξ and χ are three-vectors. Such a one-particle state $|\alpha\rangle$ characterized by the parameters $\alpha = (\xi, \chi, \sigma)$ is created from the $|0\rangle$ vacuum state by $\hat{a}^{\dagger}$, the following wave-packet creation operator:

$$\hat{\alpha}^{\dagger} = \int \frac{\mathrm{d}^3 x}{\left(\sigma\sqrt{\pi}\right)^{3/2}} \exp\left\{ -\frac{1}{2\sigma^2}\left(p - \chi\right)^2 - i\xi\left(p - \chi\right) \right\} \hat{a}^{\dagger}(p), \tag{1}$$

where $\hat{a}^{\dagger}(p)$ is the usual pion creation operator. The normalization of the states $|\alpha\rangle$ is $\langle\alpha|\alpha\rangle = 1$, while states with different parameters have overlap, which we denote by $\langle\alpha_i|\alpha_j\rangle := \gamma_{ij}$. With this, the properly normalized n-particle states are

$$|\alpha_1 \ldots \alpha_n\rangle = \left(\sum_{\sigma} \prod_{i=1}^{n} \gamma_{i\sigma_i} \right)^{-1/2} \prod_{i=1}^{n} \hat{\alpha}_i^{\dagger} |0\rangle, \tag{2}$$

where the summation runs over all the possible σ permutations of the n indices, and σ_i means the image of the i index by the permutation σ. We note the expression of γ_{ij} (if $\sigma_i = \sigma_j = \sigma$):

$$\gamma_{ij} = \exp\left\{ -\frac{\sigma^2\left(\xi_i - \xi_j\right)^2}{4} - \frac{\left(\chi_i - \chi_j\right)^2}{4\sigma^2} + \frac{i}{2}\left(\chi_i - \chi_j\right)\left(\xi_i + \xi_j\right) \right\}. \tag{3}$$

The investigated model is defined through its density matrix $\hat{\rho}$ as

$$\hat{\rho} = \sum_{n=0}^{\infty} p_n \hat{\rho}_n \quad , \quad \mathrm{Tr}\hat{\rho} = 1 \tag{4}$$

with p_n being the multiplicity distribution, and $\hat{\rho}_n$ the density matrices of events with fixed number of particles. The normalization is as usual: $\mathrm{Tr}\hat{\rho}_n = 1$ for any n, so $\sum_{n=0}^{\infty} p_n = 1$. The $\hat{\rho}_n$ matrices are built up of the multi-particle states defined above:

$$\hat{\rho}_n = \int \mathrm{d}\alpha_1 \ldots \mathrm{d}\alpha_n \rho\left(\alpha_1 \ldots \alpha_n\right) |\alpha_1 \ldots \alpha_n\rangle\langle\alpha_1 \ldots \alpha_n|, \tag{5}$$

where $\rho\left(\alpha_1 \ldots \alpha_n\right)$ are density functions. There is a special type of them, in what case the model is analytically solvable [4], namely, if the normalization factor in Eq. (2) is cancelled:

$$\rho_n\left(\alpha_1 \ldots \alpha_n\right) = \frac{1}{\mathcal{N}(n)} \left(\sum_{\sigma} \prod_{j=1}^{n} \gamma_{j\sigma_j} \right) \prod_{i=1}^{n} \rho_1(\alpha). \tag{6}$$

Finally, the one-particle densities $\rho_1(\alpha)$ are assumed to be Gaussian:

$$\rho_1(\alpha_1) = \delta(\sigma_1 - \sigma) \times \frac{1}{(2\pi R^2)^{3/2}} \exp\left\{-\frac{\xi_1^2}{2R^2}\right\} \times \frac{1}{(2\pi mT)^{3/2}} \exp\left\{-\frac{\chi_1^2}{2mT}\right\} \tag{7}$$

with T being the temperature and R the radius. The normalization factor in Eq. (6) is

$$\mathcal{N}(n) = \sum_\sigma \prod_{i=1}^n \int \mathrm{d}\alpha_i \, \rho_1(\alpha_i)\gamma_{i\sigma_i}. \tag{8}$$

The definition of the model is completed with the specification of the multiplicity distribution p_n, which step is postponed to Section 6.

3 Observables

We want to calculate inclusive n-particle distributions as well as exclusive ones (considering events only with fixed m number of particles). The definitions of the exclusive and inclusive quantities are, respectively

$$N_i^{(m)}(k_1,\ldots,k_n) = \mathrm{Tr}\left\{\hat{\rho}_m \hat{a}^\dagger(k_1)\hat{a}(k_1)\ldots\hat{a}^\dagger(k_n)\hat{a}(k_n)\right\}, \tag{9}$$

$$N_i(k_1,\ldots,k_n) = \mathrm{Tr}\left\{\hat{\rho}\hat{a}^\dagger(k_1)\hat{a}(k_1)\ldots\hat{a}^\dagger(k_n)\hat{a}(k_n)\right\} = \sum_{m=i}^\infty p_m N_i^{(m)}(k_1,\ldots,k_n). \tag{10}$$

The normalizations are

$$\int \mathrm{d}^3 k \, N_i^{(m)}(k, k_1,\ldots,k_{i-1}) = (m-i+1) N_{i-1}^{(m)}(k_1,\ldots,k_{i-1}), \tag{11}$$

$$\int \mathrm{d}^3 k_1 \ldots \mathrm{d}^3 k_i \quad N_i^{(m)}(k_1,\ldots,k_i) \quad = \frac{m!}{(m-i)!}, \tag{12}$$

$$\int \mathrm{d}^3 k_1 \ldots \mathrm{d}^3 k_i \quad N_i(k_1,\ldots,k_i) \quad = \langle n(n-1)\ldots(n-i+1)\rangle, \tag{13}$$

where $\langle (n)\ldots(n-i+1)\rangle$ is the i-th factorial moment of the multiplicity distribution. Because of the special choice of the density functions in eq. (6) in the present model it is possible to write down a relatively simple expression for the exclusive distributions with the auxiliary function

$$\overline{\rho}(p,q) = \int \mathrm{d}\alpha \, \langle \alpha|p\rangle \rho_1(\alpha)\langle q|\alpha\rangle. \tag{14}$$

Performing some combinatorics one finds that

$$N_n^{(n)}(k_1,\ldots,k_n) = \frac{n!}{\mathcal{N}(n)} \sum_\sigma \prod_{i=1}^n \overline{\rho}(k_i, k_{\sigma_i}). \tag{15}$$

From this one obtains the lower order exclusive functions as

$$N_m^{(n)}(k_1,\ldots,k_n) = \frac{n!}{(n-m)!} \frac{1}{\mathcal{N}(n)} \times \int \mathrm{d}^3 k_{m+1}\ldots\mathrm{d}^3 k_n \sum_\sigma \prod_{i=1}^n \overline{\rho}(k_i, k_{\sigma_i}). \tag{16}$$

For sake of clarity we write down the form of the two lowest order functions:

$$N_1^{(n)}(k_1) = \frac{n}{\mathcal{N}(n)} \int \mathrm{d}^3 k_2 \ldots \mathrm{d}^3 k_n \sum_\sigma \prod_{i=1}^n \overline{\rho}(k_i, k_{\sigma_i}), \tag{17}$$

$$N_2^{(n)}(k_1, k_2) = \frac{n(n-1)}{\mathcal{N}(n)} \int \mathrm{d}^3 k_3 \ldots \mathrm{d}^3 k_n \sum_\sigma \prod_{i=1}^n \overline{\rho}(k_i, k_{\sigma_i}). \tag{18}$$

The calculation of these $n!$-termed integrals is usually a NP-hard problem. However, in the present model a nice analytic solution is possible by means of a so-called ,,ring algebera" [6]. The generalization to the case of wave-packets is found in Ref. [4], in the next section we summarize the results.

4 Analytic solutions

The integrals in eqs. (17) and (18) can be evaluated using Wick's theorem: the $\overline{\rho}$ functions in a term of te sum over the permutations can be ordered to rings (i.e. orbits of the given permutation). The evaluation of the integrals then reduces to the evaluation of this so-called ring algebra. We just quote the results: one introduces the set of functions $G_i(p, q)$ as

$$G_1(p, q) = \overline{\rho}(p, q) \quad , \quad G_n(p, q) = \int d^3 p_1 \overline{\rho}(p, p_1) G_{n-1}(p_1, q). \tag{19}$$

These functions appear as ,,ring integrals". Another important quantities are the normalization constants $\mathcal{N}(n)$. Following Ref. [4] (but slightly deviating from its conventions), we use an alternative notation as $\omega_n = \frac{1}{n!}\mathcal{N}(n)$. Putting all this together, the one- and two-particle exclusive distribution functions are given by

$$N_1^{(n)}(k_1) = \frac{1}{\omega_n} \sum_{i=1}^n G_i(k_1, k_1)\omega_{n-i}, \tag{20}$$

$$N_2^{(n)}(k_1, k_2) = \frac{1}{\omega_n} \sum_{i=2}^n \sum_{l=1}^{i-1} \{G_l(k_1, k_1)G_{i-l}(k_2, k_2) + G_l(k_1, k_2)G_{i-l}(k_2, k_1)\} \omega_{n-i}. \tag{21}$$

The general $N_i^{(n)}$ could be expressed in a much similar way. The functions $G_n(p, q)$ are found to have an analytic form [4]. The key point is the stability property of the Gaussian distribution: the wave-packets as well as the density functions in eqs. (5) and (6) are Gaussians, so the $G_n(p, q)$ functions will be also Gaussian, so we parametrize them as

$$G_i(p, q) = h_i \exp\left\{-a_i\left(p^2 + q^2\right) + g_i pq\right\}, \tag{22}$$

and then from the recursive definition in eq. (19) one can derive the recurrence relations, which determine the quantities h_i, a_i and g_i:

$$h_{n+1} = h_1 h_n \left(\frac{\pi}{a_1 + a_n}\right)^{\frac{3}{2}}, \; h_1 = \frac{1}{\left(\pi\sigma_T^2\right)^{3/2}}, \tag{23}$$

$$a_{n+1} = a_1 - \frac{g_1^2}{4\left(a_1 + a_n\right)}, \; a_1 = \frac{1}{2\sigma_T^2} + \frac{R_{eff}^2}{2}, \tag{24}$$

$$g_{n+1} = \frac{g_1 g_n}{2\left(a_1 + a_n\right)}, \; g_1 = R_{eff}^2. \tag{25}$$

The constants σ_T and R_{eff} in the initial values can be considered as effective values of the source parameters:

$$\sigma_T^2 = \sigma^2 + 2mT \quad , \quad R_{eff}^2 = R^2 + \frac{mT}{\sigma^2 \sigma_T^2}. \tag{26}$$

The solution of the recurrence relations is given by the following equations:

$$a_n = \frac{g_1}{2}\frac{Y_{n+1}}{Y_n} - a_1 \quad , \quad h_n = h_1^n \left[\left(\frac{2\pi}{g_1}\right)^n \frac{1}{Y_n}\right]^{3/2} \quad , \quad g_n = \frac{g_1}{Y_n}, \tag{27}$$

where the auxiliary quantities are

$$Y_n = \frac{r_+^n - r_-^n}{r_+ - r_-} \quad , \quad r_\pm = \frac{2a_1}{g_1} \pm \sqrt{\left(\frac{2a_1}{g_1}\right)^2 - 1}. \tag{28}$$

These quantities give the auxiliary G_i functions. We do not go into the details of the derivation, just recommend Ref. [4] for a thorough discussion. We should note that the definitions and equations used in the present paper differ slightly from those of Refs. [4,5], as the multiplicity distribution is introduced here in the last possible step.

For the exclusive spectra one needs the ω_n quantities. They obey the following relation, obtained from eq. (20) using the normalization condition of $N_1(k)$:

$$\omega_n = \frac{1}{n} \sum_{i=1}^{n} \omega_{n-i} i C_i \quad , \quad C_n = \int \mathrm{d}k G_n(k,k). \tag{29}$$

The solution to this can be obtained from the expansion of the following power series:

$$\tilde{g}(z) := \sum_{n=0}^{\infty} \omega_n z^n = \exp\left(\sum_{n=1}^{\infty} C_n z^n\right). \tag{30}$$

We do not go in details now, just mention that in two limiting cases the ω_n quantities have a simple form: if $R_{eff}^2 \sigma_T^2 \to \infty$ (that is, in the rare gas limit)

$$C_n = \delta_{n,1} \quad \Rightarrow \quad \omega_n = \frac{1}{n!}, \tag{31}$$

while in the opposite, dense gas limiting case (that is, $R_{eff}^2 \sigma_T^2 \to 0$)

$$C_n = \frac{1}{n} \quad \Rightarrow \quad \omega_n = 1. \tag{32}$$

5 Inclusive spectra

In a real experimental situation one usually measures the inclusive spectra, exclusive measurements can be done much harder. However, for the inclusive spectra ne needs the exclusive ones, and so one needs to specify the multiplicity distribution, and at this point difficulties occur.

We could specify any kind of multiplicity distribution, but eq. (30) suggests a way that enables us to continue the calculation analytically. Namely, the role of the C_n-s in eq. (29) is very similar than that of the so called combinants of a multiplicity distribution: in this sense ω_n corresponds to the multiplicity distribution itself. So one reasonable possibility is to choose p_n as

$$p_n = \left(\sum_{m=0}^{\infty} \omega_m \overline{n}_0^m\right)^{-1} \omega_n \overline{n}_0^n. \tag{33}$$

where $\overline{n}_0$ is some ,,mean" value. We note two important relations concerning infinite sums:

$$\left(\sum_{k=0}^{\infty} a_k\right)\left(\sum_{l=1}^{\infty} b_l\right) = \sum_{k'=1}^{\infty} \left(\sum_{l'=1}^{\infty} a_{k'-l'} b_{l'}\right), \tag{34}$$

$$\left(\sum_{k=0}^{\infty} a_k\right)\left(\sum_{l=1}^{\infty} b_l\right)\left(\sum_{q=1}^{\infty} b_q\right) = \sum_{k'=2}^{\infty} \left(\sum_{l'=2}^{\infty} \sum_{q'=1}^{\infty} a_{k'-l'} b_{l'-q'} c_{q'}\right). \tag{35}$$

Using these, the inclusive spectra are found to have a simple form [4]. Introducing the function

$$G(p,q) = \sum_{n=0}^{\infty} G_n(p,q)\,\overline{n}_0^n, \tag{36}$$

one has

$$N_1(k_1) = G(k_1, k_1), \tag{37}$$
$$N_2(k_1, k_2) = G(k_1, k_1)\,G(k_2, k_2) + G(k_1, k_2)\,G(k_2, k_1). \tag{38}$$

Higher order functions can be obtained in a much similar way. It is important to note two specialities of these expressions. First, since $G(p,q)$ has an analytic form, all the measurable inclusive distribution functions can be calculated in an easy way. Second, evaluating $G(p,q)$ at the intercept point $p = q$, we get $G(p,p) = 1$, so, for example, if we define the two-particle correlation function as

$$C(p,q) := \frac{N_2(p,q)}{N_1(p)\,N_1(q)}, \tag{39}$$

we see, that it has a physical intercept behavior, $C(p,p) = 2$. Similarly, for the higher-order correlation functions, $C_n(p, ..., p) = n!$.

This multiplicity distribution was considered in Refs. [4,5]. Our aim was to generalize the multiplicity distribution and find analytic expressions for the inclusive spectra. A motivation arises from the limiting cases of the present multiplicity distribution: we see from eqs. (31) and (32) that in the rare gas limiting case p_n approaches a Poissonian distribution with $\overline{n}_0$ mean, and in the opposite, dense gas case we have

$$p_n = \frac{\langle n \rangle^n}{(\langle n \rangle + 1)^{n+1}} \quad , \quad \langle n \rangle = \frac{\overline{n}_0}{1 - \overline{n}}. \tag{40}$$

We would be interested in a distribution which has the opposite behavior: in the rare gas limit it should be a thermal, negative binomial distribution, and in the dense limit a condensed, laser-like Poissonian distribution. We will see that this goal can be achieved without difficulties by redefining p_n, but the analytic simplicity will be lost.

6 General p_n-s, uniqueness

We can generalize the definition of p_n in eq. (33) to a case of an integral transformation as

$$p_n = \int_0^{\infty} \mathrm{d}y \left(\sum_{m=0}^{\infty} \omega_m y^m \right)^{-1} \omega_n y^n H(y), \tag{41}$$

This is a pretty general multiplicity distribution, since the function $H(y)$ is arbitrary. If $H(y) = \delta(y - \overline{n}_0)$, then p_n is the original one as in eq. (33). In the rare gas limiting case the transformation is just

$$p_n = \int_0^{\infty} \mathrm{d}y \exp(-y) \frac{y^n}{n!} H(y), \tag{42}$$

which is the so-called Poissonian transformation of the $H(y)$ function. As an example, let us consider the function $H(y) = \frac{1}{\langle n \rangle} \exp\left(-\frac{y}{\langle n \rangle}\right)$: it yields a geometrical distribution in the rare gas limiting case:

$$H(y) = \frac{1}{\langle n \rangle} \exp\left(-\frac{y}{\langle n \rangle}\right) \quad \Rightarrow \quad p_n = \frac{\langle n \rangle^n}{(\langle n \rangle + 1)^{n+1}}. \tag{43}$$

Another motivation for the choice of the multiplicity distribution as in eq. (41) lies in the calculation method of the inclusive spectra. Before going into this, let us mention, that if we allow $H(y)$ to depend not only on y, but also on the model parameters (P_{eff}^2 and σ_T^2), then practically all type of multiplicity distributions can be expressed as an integral transform according to eq. (41). So for a given multiplicity distribution one can find the appropriate $H(y)$ function.

We can use eqs. (34) and (35) and their analogues with more variables to calculate the inclusive spectra. For example, from eqs. (10) and (34) $N_1(k_1)$ is

$$N_1(k_1) = \int_0^\infty dy \frac{H(y)}{\sum_{m=0}^\infty \omega_m y^m} \sum_{i=1}^\infty y^i \sum_{j=1}^i G_j(k_1, k_1) \omega_{i-j} = \sum_{n=1}^\infty G_n(k_1, k_1) f_n, \qquad (44)$$

where $f_n = \int_0^\infty H(y) y^n$. In the same way, we have for the two-particle inclusive function

$$N_2(k_1, k_2) = \sum_{i=0}^\infty \sum_{j=0}^\infty \{G_i(k_1, k_1) G_j(k_2, k_2) + G_i(k_1, k_2) G_j(k_2, k_1)\} f_{i+j}, \qquad (45)$$

and so on for $N_n(k_1, \ldots, k_n)$.

We can now draw two conclusions

1. If and only if $f_{n+m} = f_n f_m$, then we can introduce the quantity $\tilde{G}(p, q) = \sum_{i=0}^\infty f_i G_i(p, q)$, and in this case the inclusive quantities have a simple analytic form: it is enough to calculate $\tilde{G}(p, q)$ *once*, and all the inclusive functions are simple combinations of it.
2. The intercept behavior of the correlation functions is physical if and only if $f_{n+m} = f_n f_m$. That is, for example, the requirement for $C_2(p, p)$ to be 2, is met only if the f_n integrals possess the mentioned relation.

But, $f_{n+m} = f_n f_m$ holds only for the case, when $H(y) = \delta(y - \overline{n}_0)$ with some $\overline{n}_0$ value. So, the conclusion of the present investigation is a negative statement: the multiplicity distribution can be generalized, but the analytic simplicity of the inclusive distribution functions as well as the physical behavior of the correlation functions do not remain for a more general p_n than investigated already in Ref. [4].

7 Summary

The aim of this analysis was to generalize the pion-laser model. The model was first described by S. Pratt [6], and then solved entirely by one of us (T. Cs.) together with J. Zimányi [4,5]. We have found, that although it is possible to modify the definition of the multiplicity distribution so that the thermal or chaotic limit corresponds to a negative binomial multiplicity distribution, but if we require the analytic solvability of the model, it uniquely prescribes that the multiplicity distribiton in the rare gas limit has to be a Poisson. This is the only case, when the intercept of the correlation function in the rare gas limit has the expected $C_2(k, k) = 2$ value. In this sense, we have proven that the original form of the pion-laser model is unique. A possible other path for analytically solvable multi-particle systems could be developed if one omits some of the assumptions of the present model: for example, if one factorizes the density function in another way than in eq. (6). But these generalizations — or more like another models — are extremely difficult to handle.

After the publication of the solution of the pion laser in refs. [4,5,6,7], a similar generalization has been proposed by Heinz, Scotto and Zhang in ref. [8]. Working with other conventions, they also obtained the general form of the inclusive spectra as in eq. (45), however, the emphasis of their work was not on the uniqueness of the analytic solution, but more on its numerical generalization. With the present work, we in a sense complete the investigations of refs. [4,5]. We find that the pion-laser model is unique, in the sense that it is the only the analytically solvable model of multi-boson symmetrization of wave-packets in the class of the considered models.

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